Hilbert-like Curves on a Hexagonal Grid and a Realization Using Crochet

2014 Joint Mathematics Meetings

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How This All Got Started
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Where It Led Next

1. Pick a pattern that has no vertices with maximal or minimal possible degree.
Where It Led Next

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Space-filling curves
Where It Led Next

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- Space-filling curves
  - Hilbert curve
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Space-filling curves

Hilbert curve
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Where It Led Next

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2. Why not work on a different tiling other than squares?
Where It Led Next

1. Pick a pattern that has no vertices with maximal or minimal possible degree.

2. Why not work on a different tiling other than squares?
Recursively Building Hexagonal Grids
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Note that there is a choice of orientation we must make.
Recursively Building Hexagonal Grids
Finding Curves That Iterate
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This isn’t as easy!
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This isn’t as easy!

- 8 ways
- 6 ways
- 10 ways
- 6 ways
Finding Curves That Iterate

This isn’t as easy!

Of all 35 possibilities, exactly zero are able to be tiled into the next iteration.

8 ways
6 ways
10 ways
6 ways
Finding Curves That Iterate

This isn’t as easy!
Finding Curves That Iterate

This isn’t as easy!

24 ways
Finding Curves That Iterate

This isn’t as easy!

24 ways ............... but only three work in the next iteration
Finding Curves That Iterate
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Summary of 2x2x2 case:

- Exactly one path iterates
- It can be used four different ways
- ...but one of those ways seems to stand out as “right”
Finding Curves That Iterate
Finding Curves That Iterate
Finding Curves That Iterate

Summary of 3x3x3 case:

- Exactly one path iterates
- It only works one way
- There are a few useful necessary conditions
Finding Curves That Iterate

Necessary conditions

- The path must end in corners that are 120° apart
- Iterates must also end 120° apart, and agree with the chosen orientation of hexagons
- Sharp (60°) corners have to be “agree”: 
Finding Curves That Iterate

Necessary conditions

• The path must end in corners that are 120° apart
• It must iterate to end the same way, and agree with the chosen orientation of hexagons
• Sharp (60°) corners have to be “agree”: 

![Image of hexagon patterns with 120° and 60° corners]
The Realization in Crochet
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References


Hexagonal & Triangular Meshes
The Standard Gosper Curve

(from Brainfilling Curves)