Hilbert-like Curves on a Hexagonal Grid and a Realization Using Crochet

2014 Joint Mathematics Meetings

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Malone University

How This All Got Started

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Where It Led Next

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Space-filling curves

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Hilbert curve

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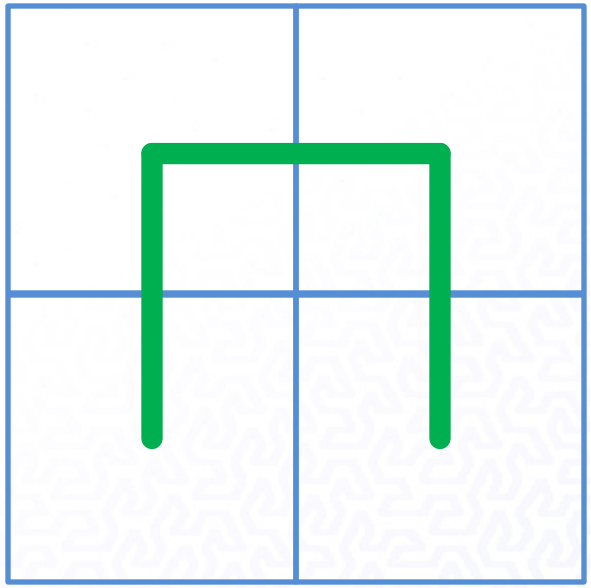

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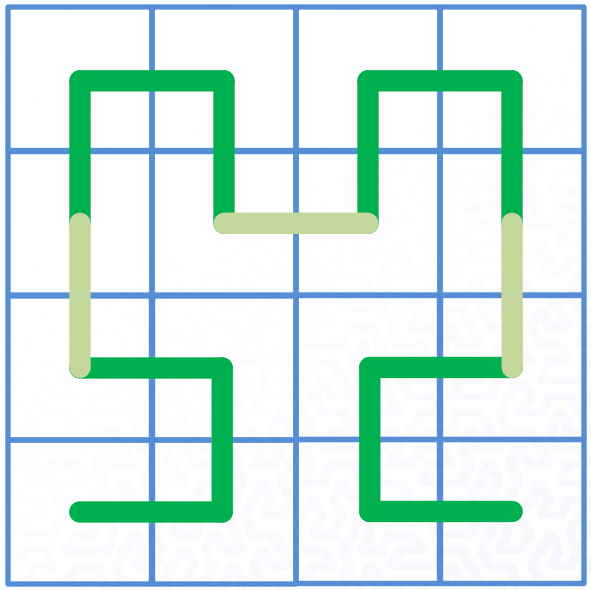

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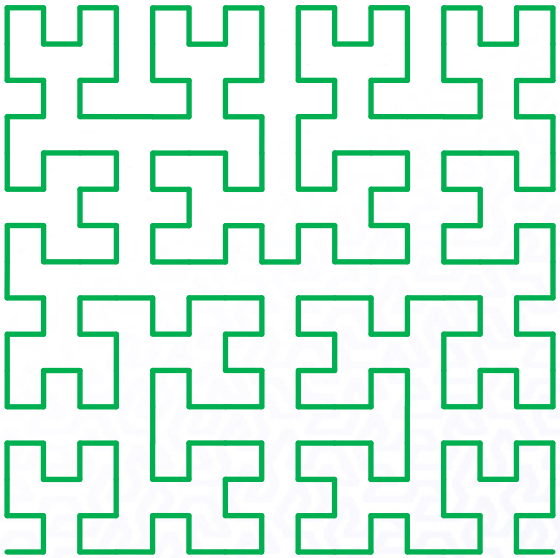

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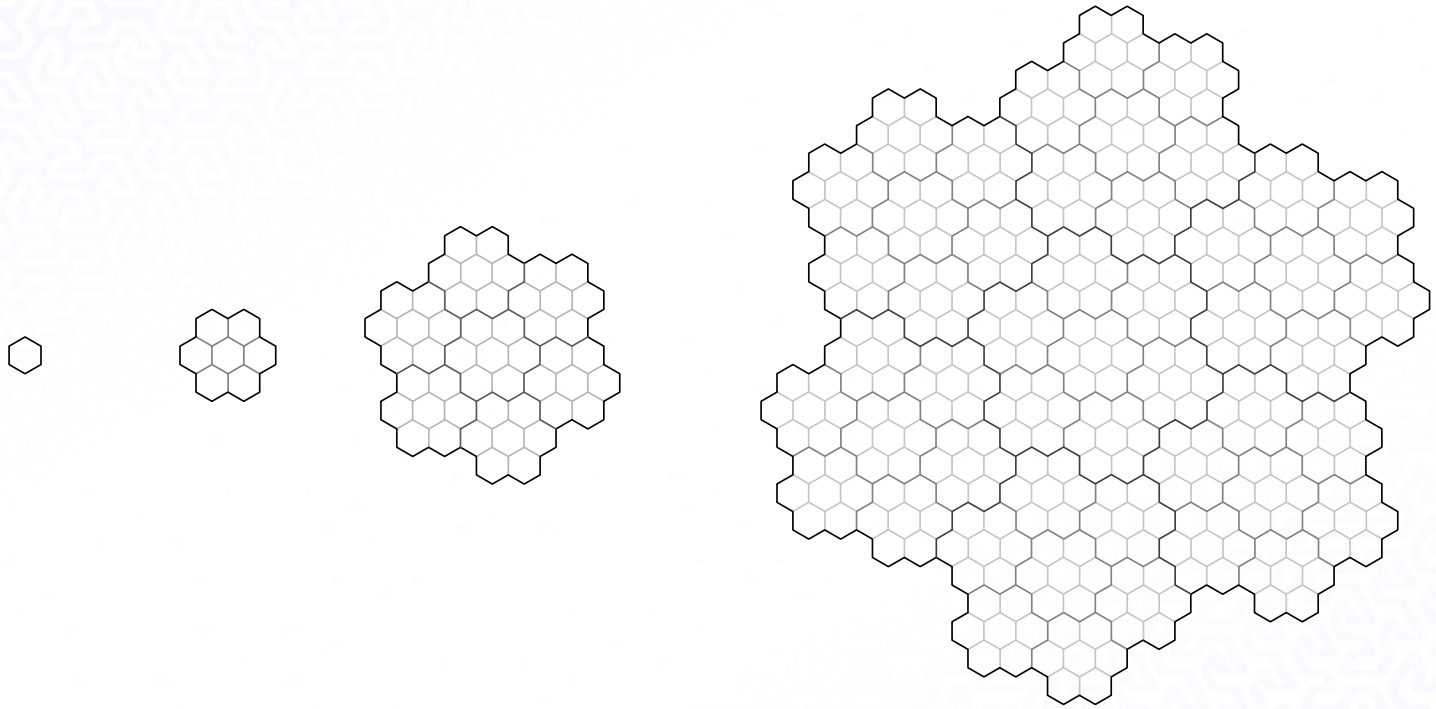
Recursively Building Hexagonal Grids



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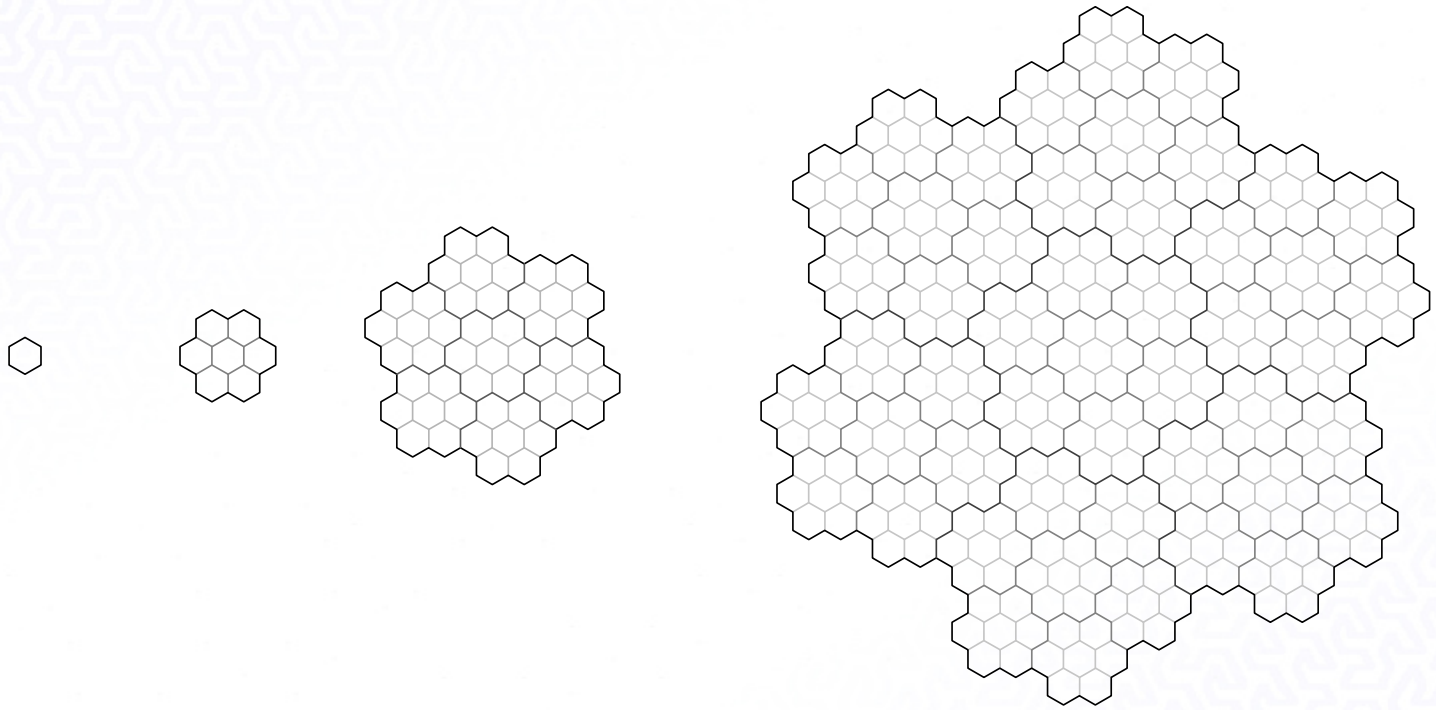
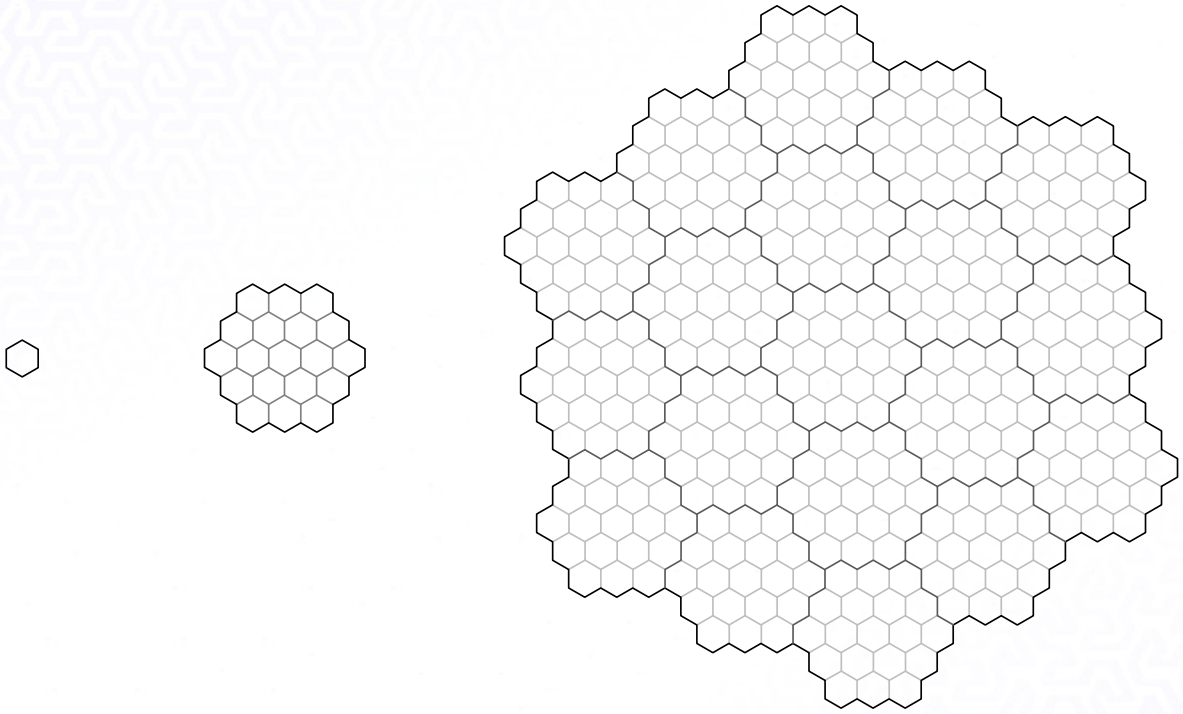


Note that there is a choice of orientation we must make.

Recursively Building Hexagonal Grids



Finding Curves That Iterate

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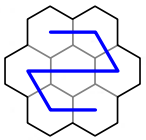

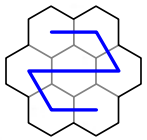

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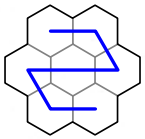

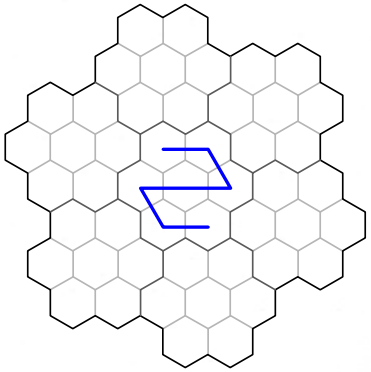

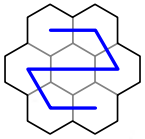

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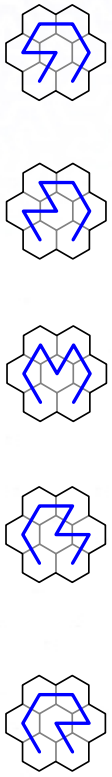

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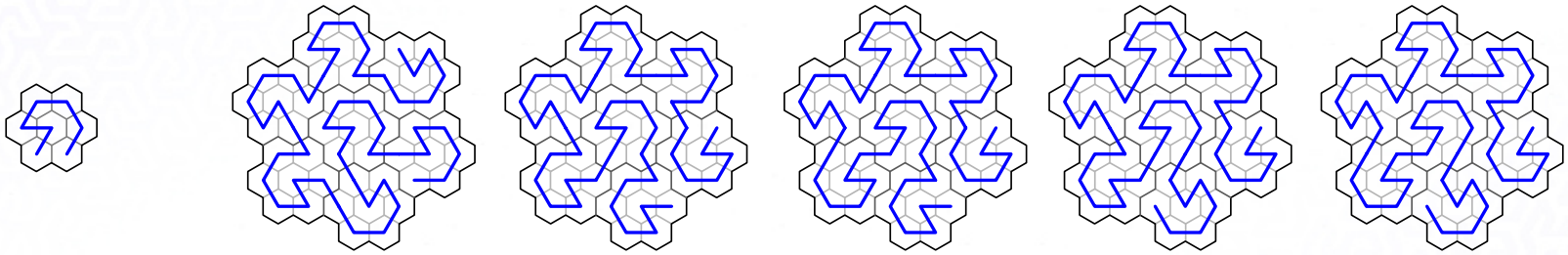

 8 ways

 6 ways

 10 ways

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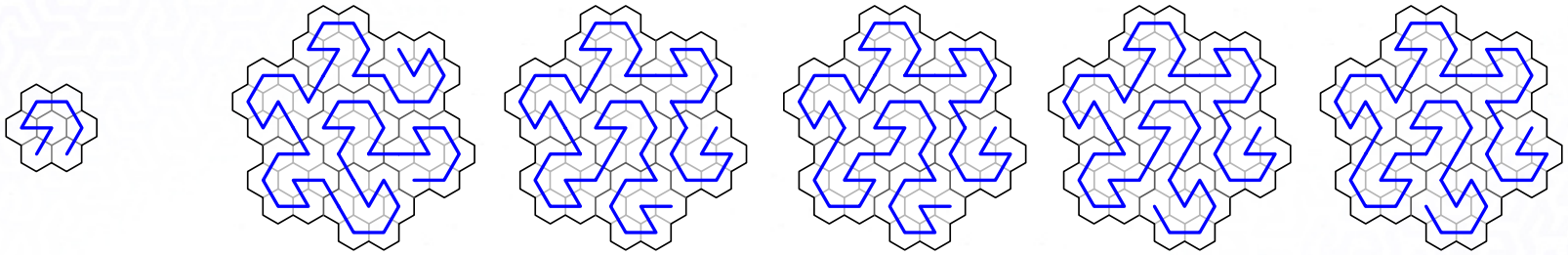


8 ways


6 ways


10 ways


6 ways

Of all 35 possibilities, exactly zero are able to be tiled into the next iteration.

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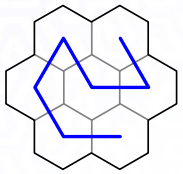

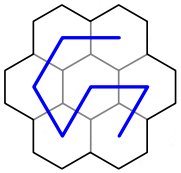

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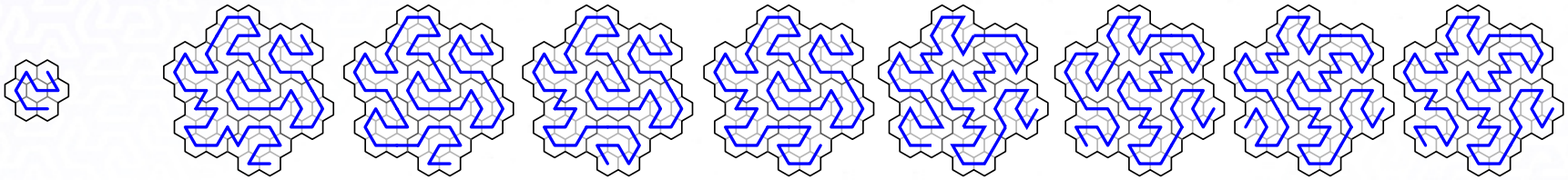

24 ways

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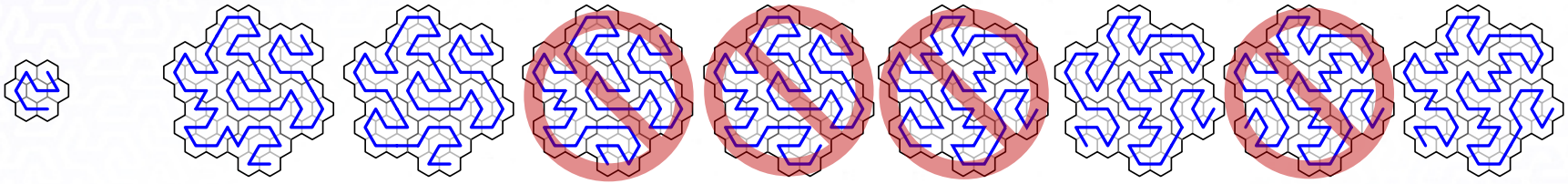

24 ways ……. but only three work in the next iteration

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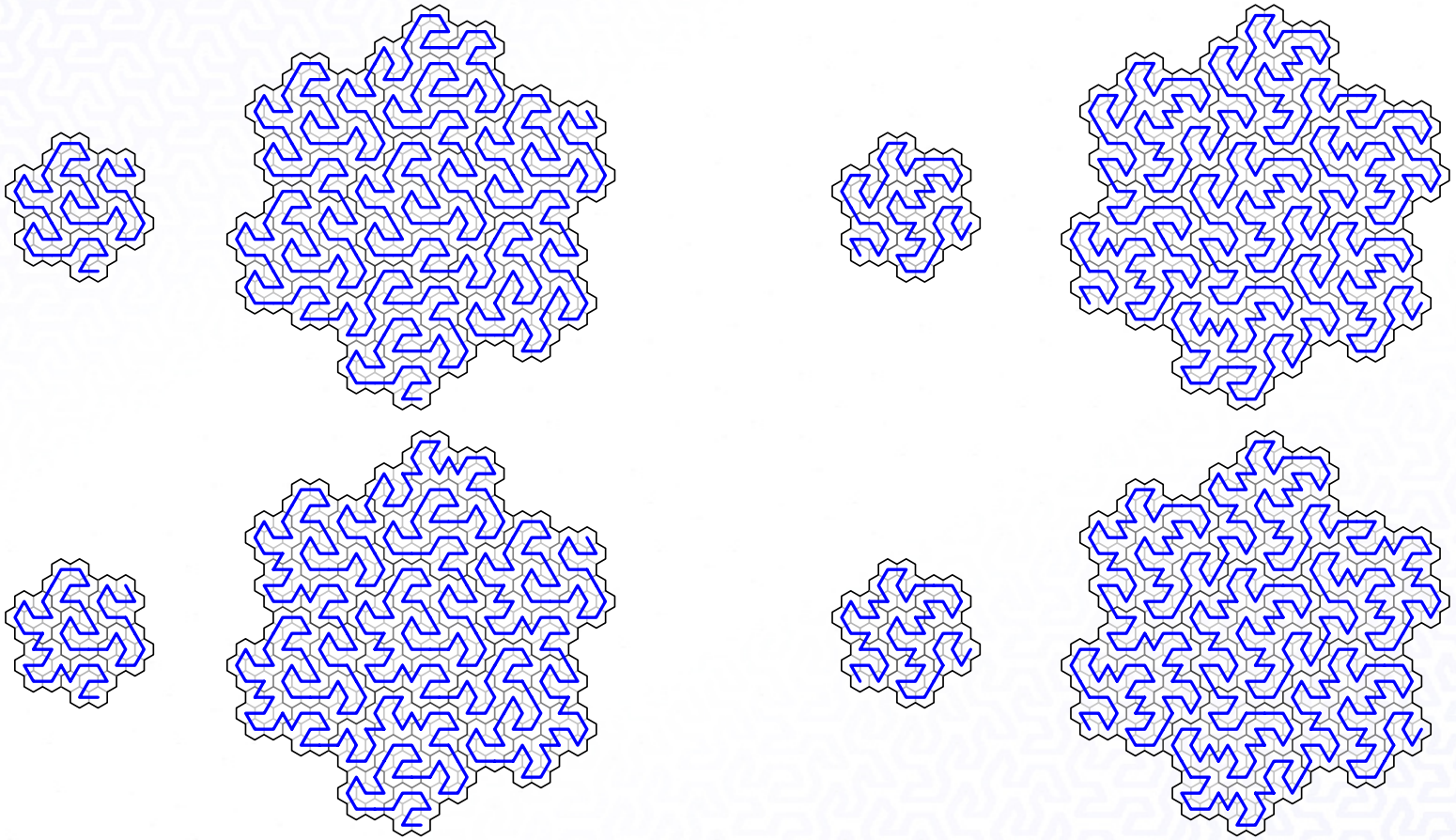

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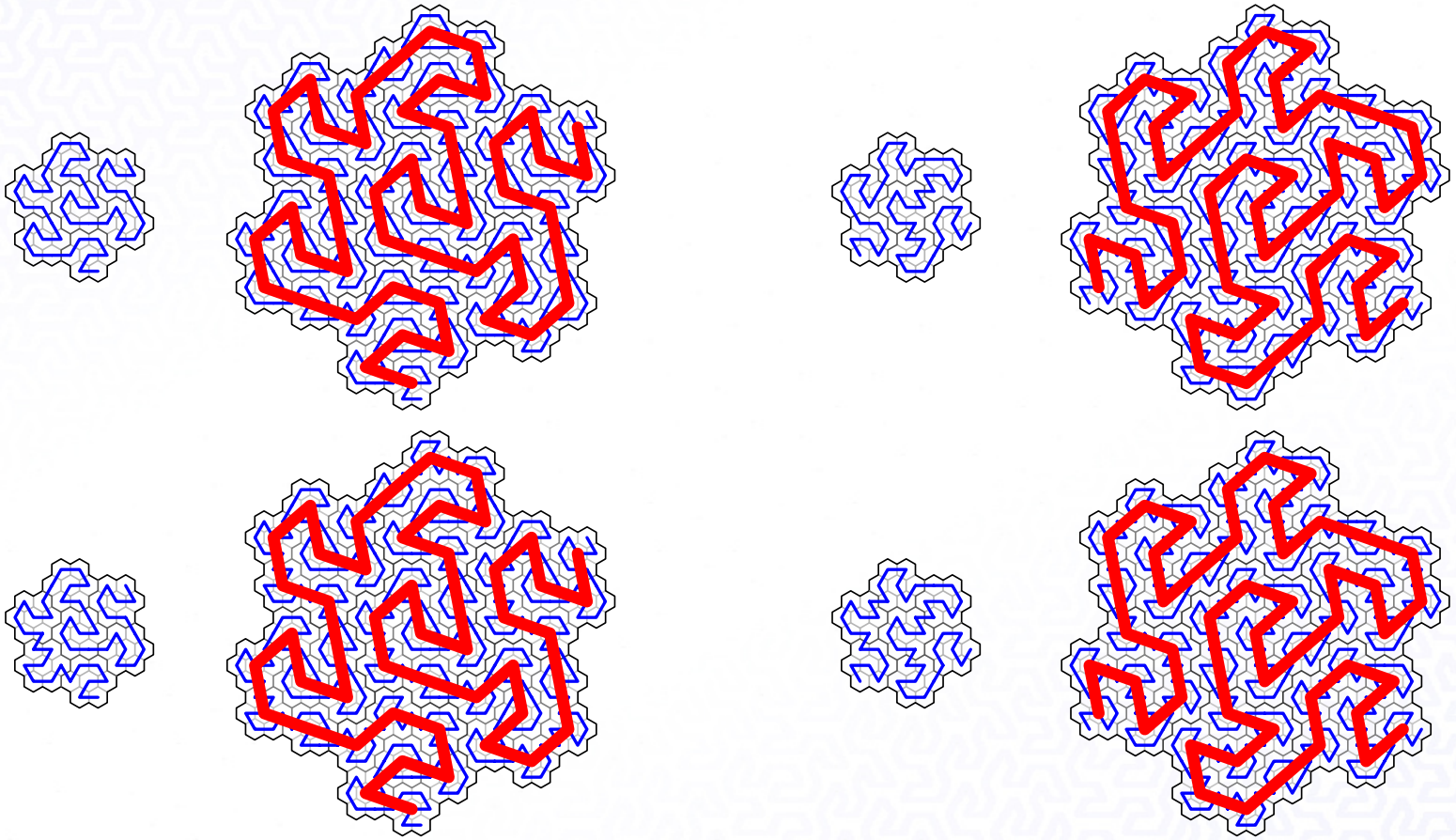

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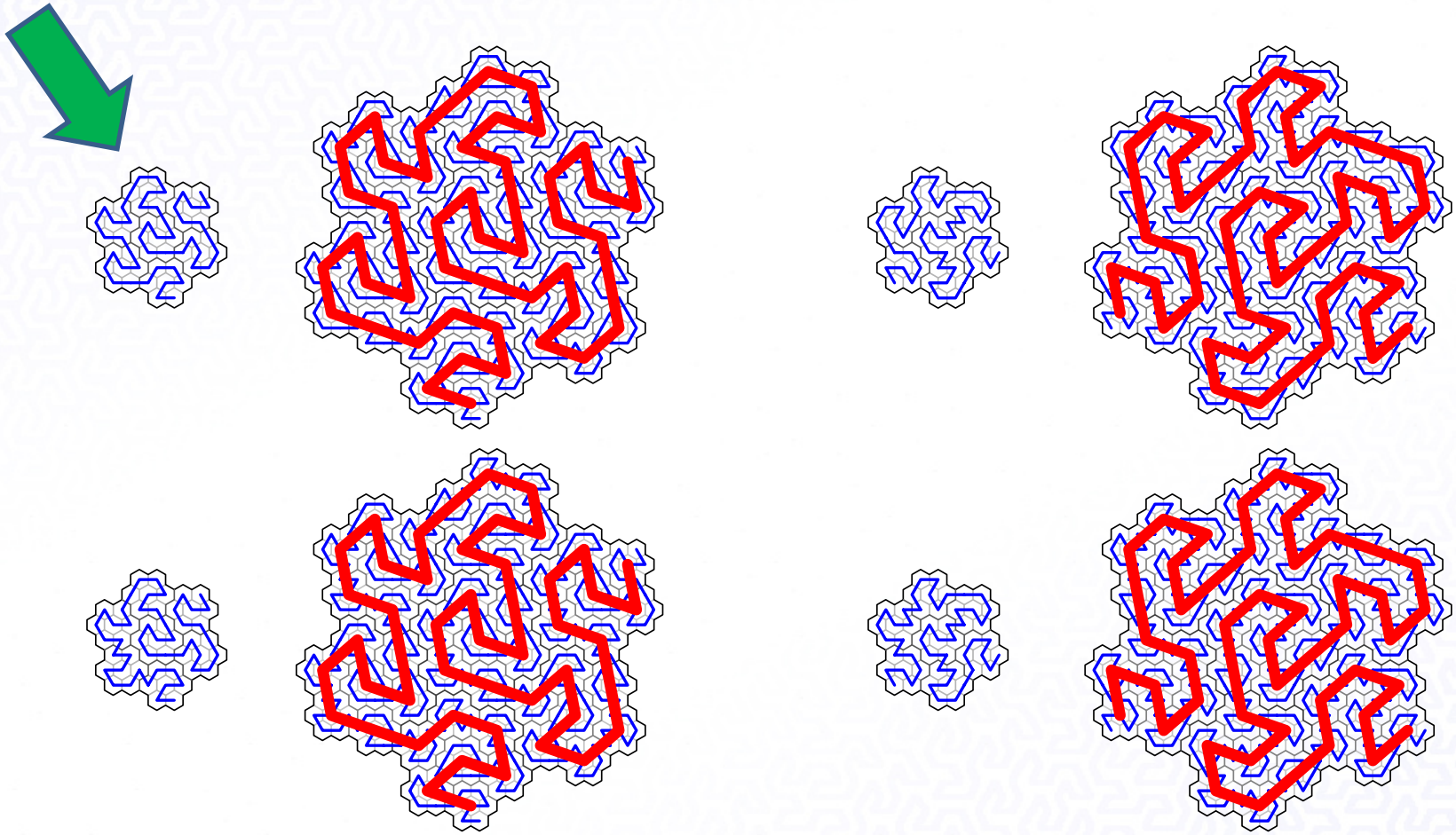

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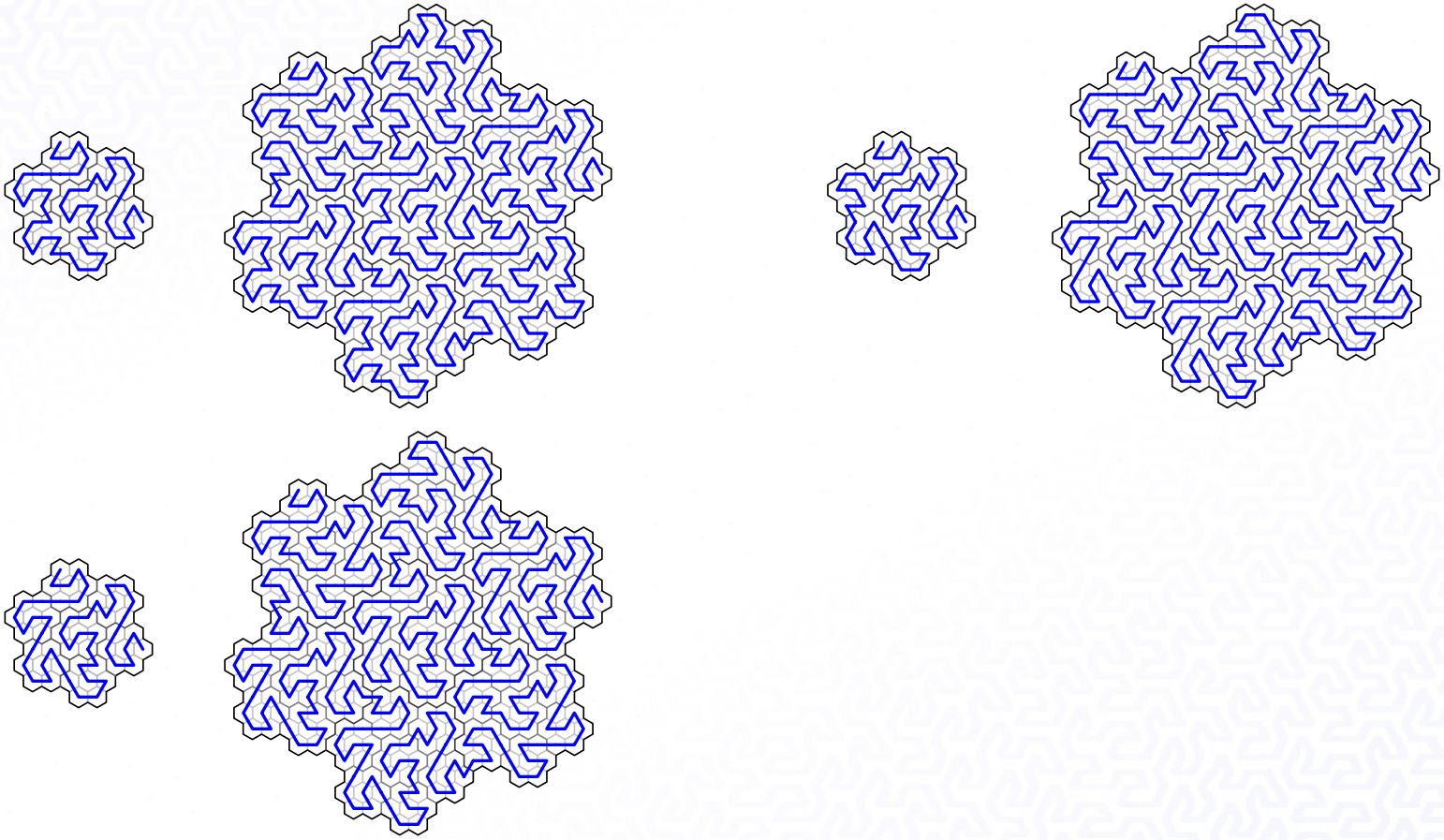

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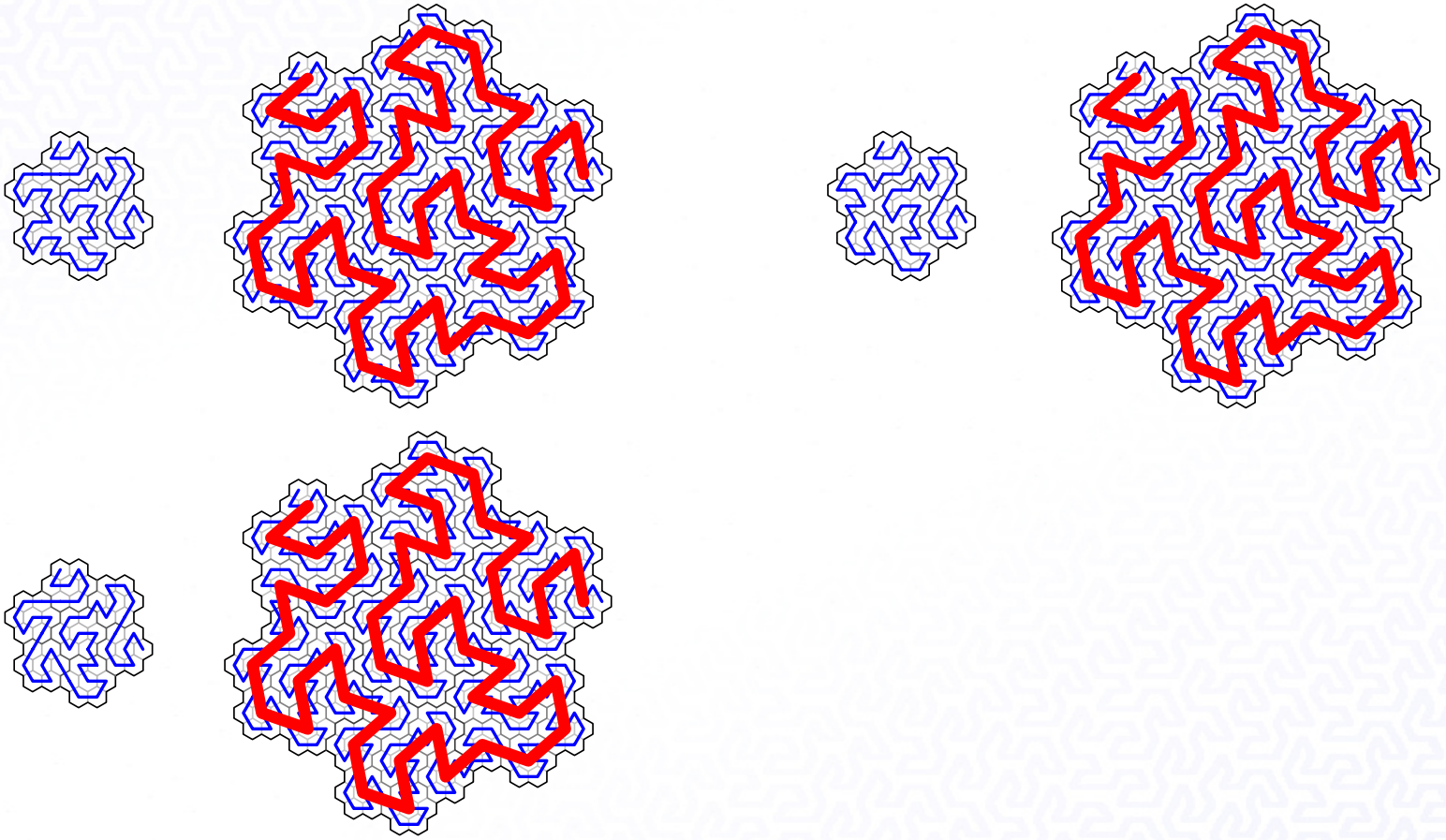
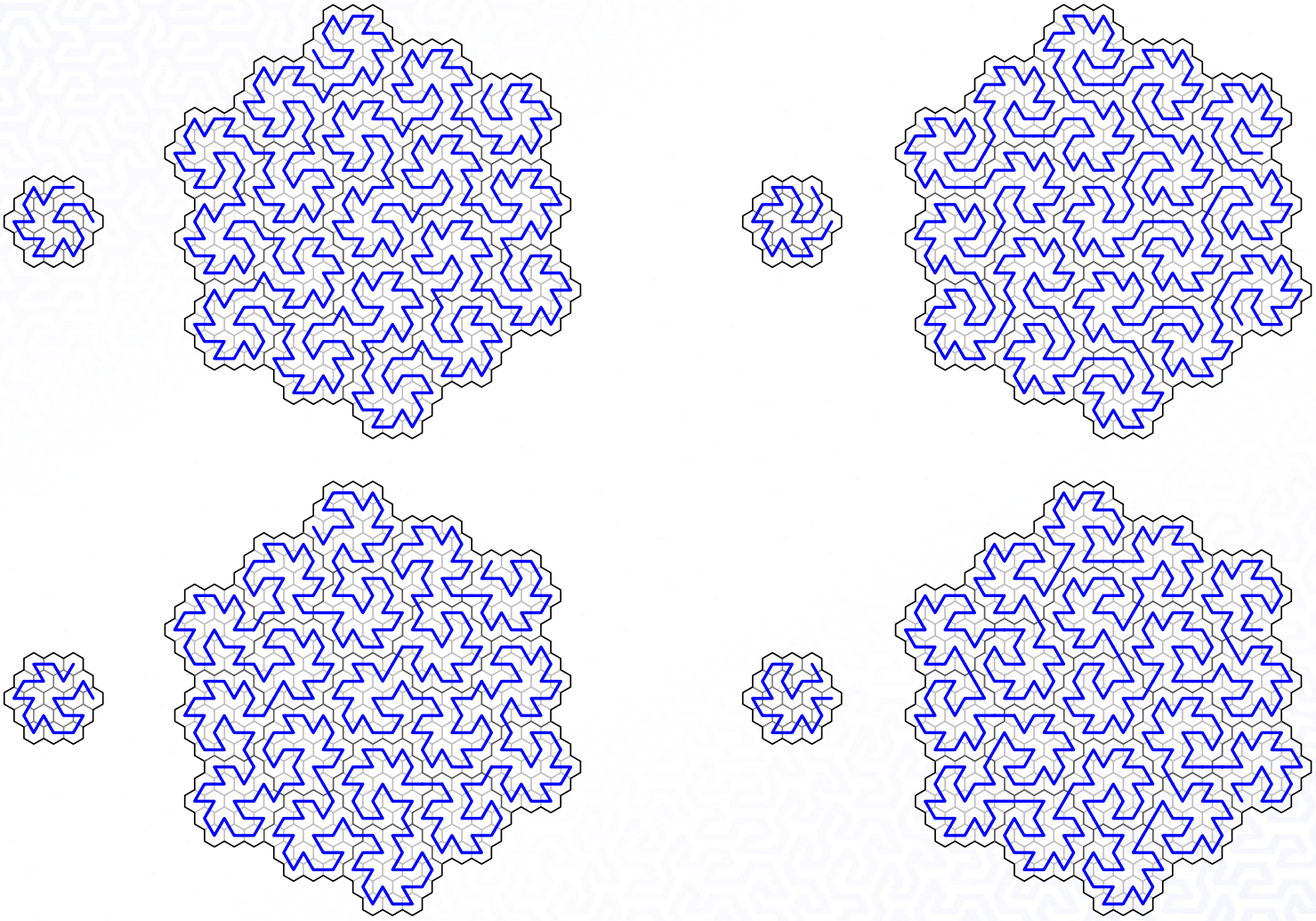


Finding Curves That Iterate

Summary of 2x2x2 case:

- Exactly one path iterates
- It can be used four different ways
- …but one of those ways seems to stand out as “right”

Finding Curves That Iterate



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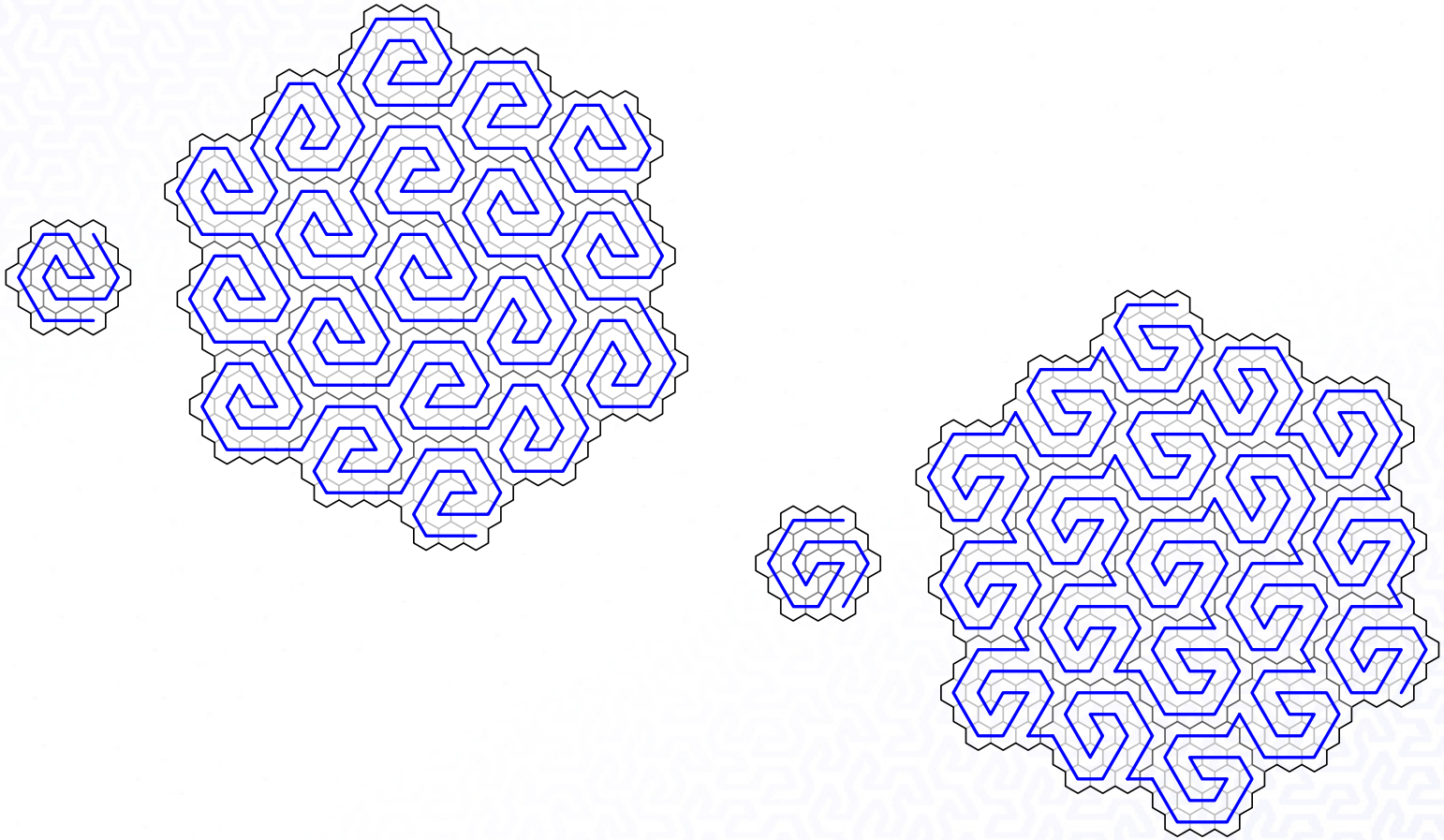


Finding Curves That Iterate

Summary of 3x3x3 case:

- Exactly one path iterates
- It only works one way
- There are a few useful necessary conditions

Finding Curves That Iterate

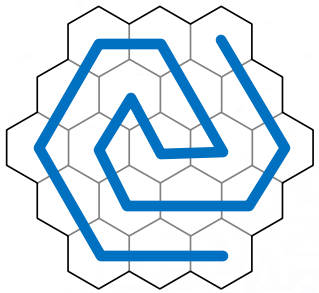
Necessary conditions

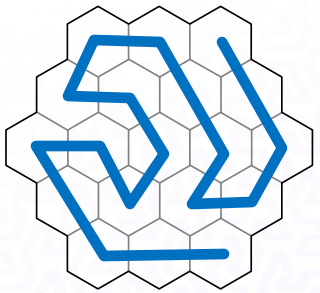
- The path must end in corners that are 120° apart
- Iterates must also end 120° apart, and agree with the chosen orientation of hexagons
- Sharp (60°) corners have to be “agree”:

Finding Curves That Iterate

Necessary conditions

- The path must end in corners that are 120° apart
- It must iterate to end the same way, and agree with the chosen orientation of hexagons
- Sharp (60°) corners have to be “agree”:





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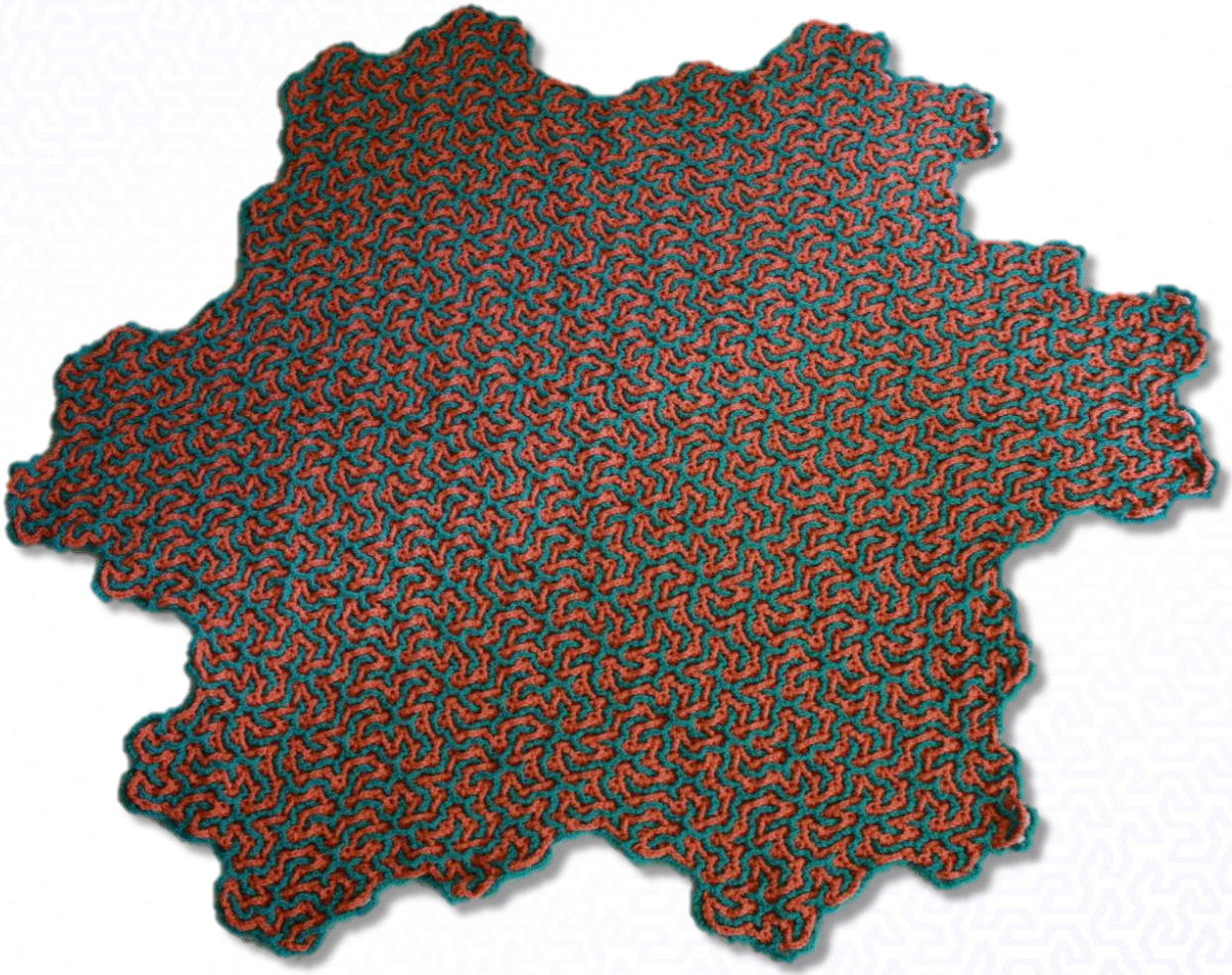

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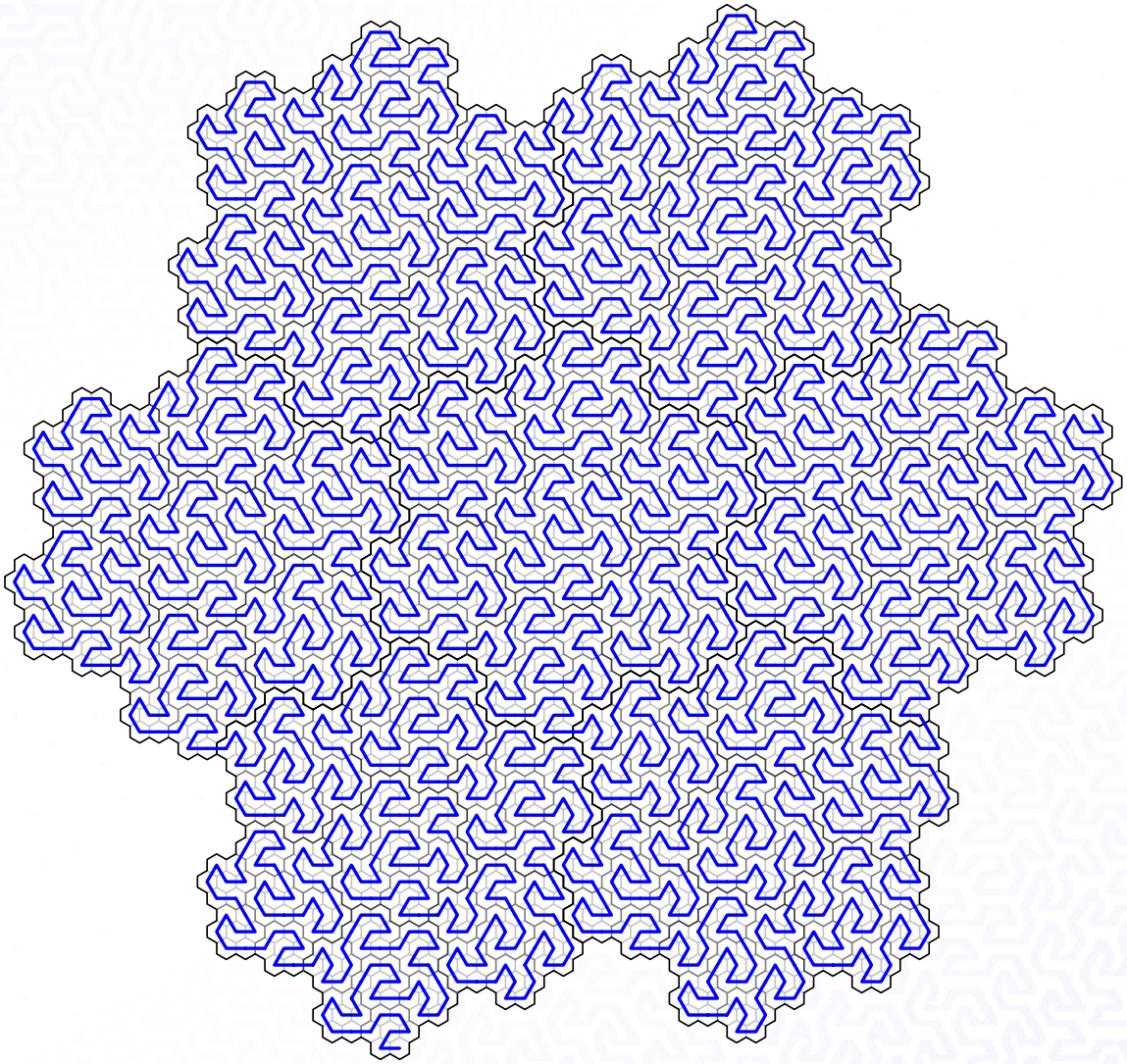

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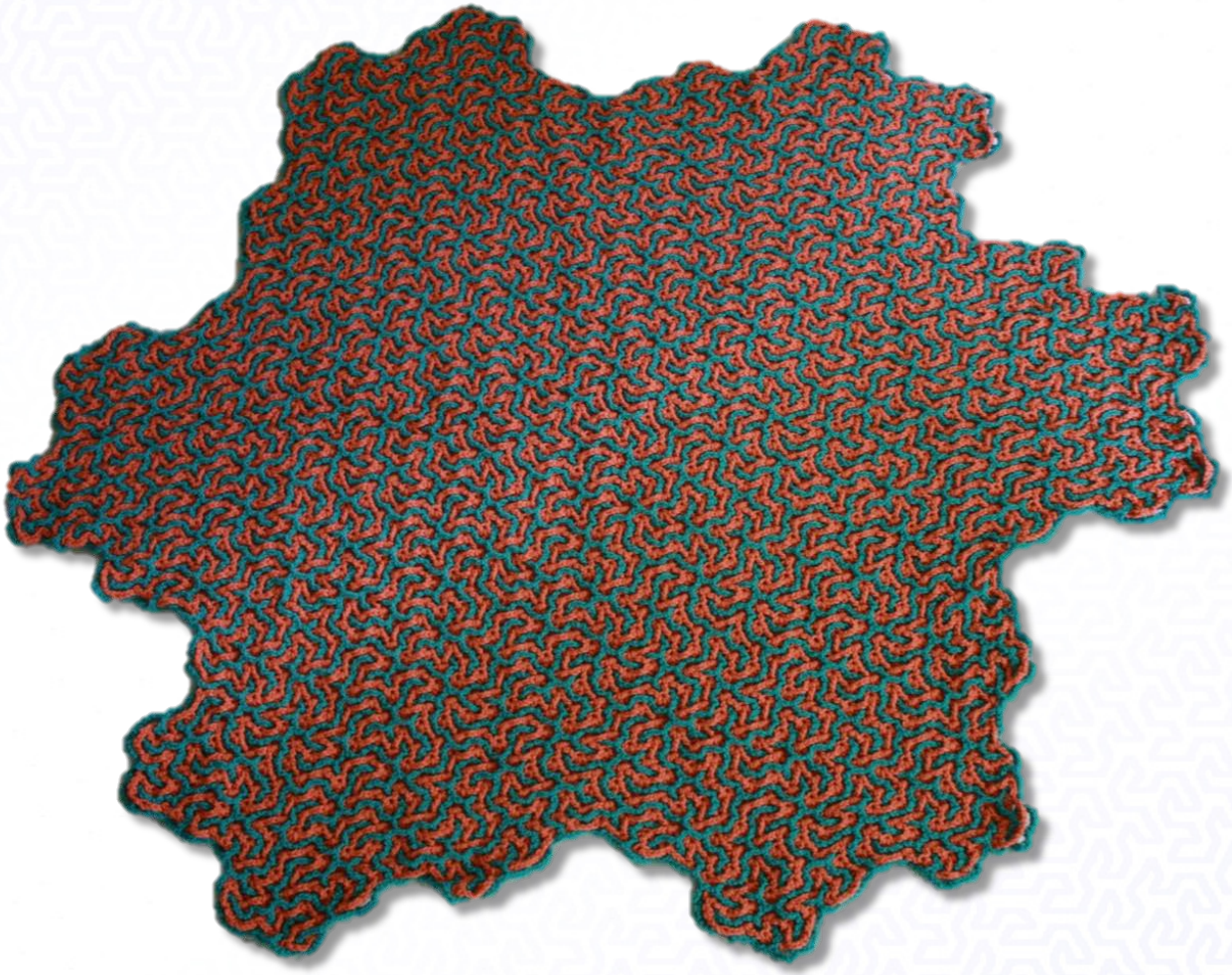


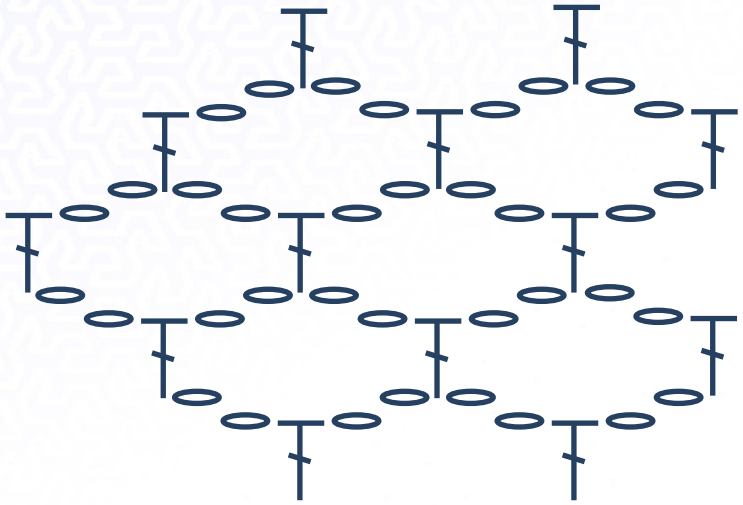
References

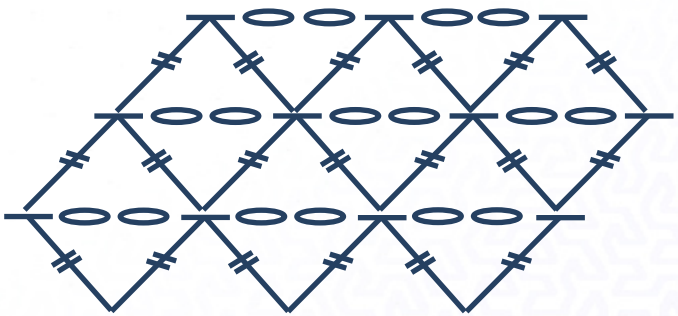
1. Akiyama, Fukuda, Ito & Nakamura “Infinite Series of Generalized Gosper Space Filling Curves”, *China-Japan Conference on Discrete Geometry, Combinatorics and Graph Theory* 2005

2. Norton, Anderson “Eighty-eight Thousand, Four Hundred and Eighteen (More) Ways to Fill Space”, *CMJ* March 2009 (v. 40, no. 2)

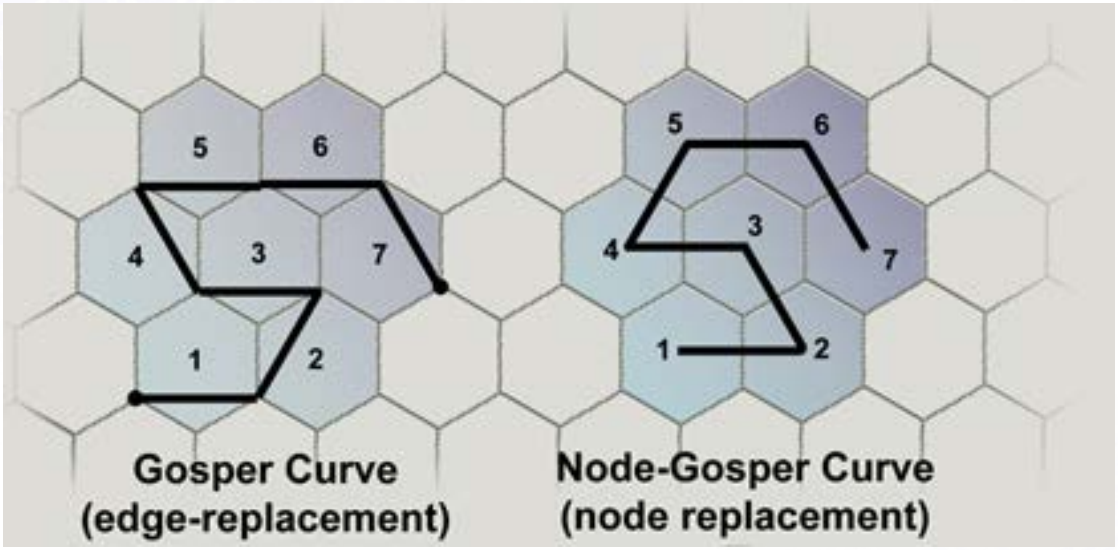
3. Ventrella, Jeffrey *Brain-filling Curves: A Fractal Bestiary*, Eyebrain Books 2012

Hexagonal & Triangular Meshes





The Standard Gosper Curve



(from Brainfilling Curves)