

Geometric Knitting II

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Recap

In “Geometric Knitting I”, I:

- ▶ discussed the relationship between knitting, topology and geometry.
- ▶ showed how to knit homogeneous spaces.
- ▶ talked about how knitting patterns relate to Riemannian metrics.

Plan

- ▶ This time I plan to give some examples of knitted mathematical objects.
- ▶ No previous knowledge of knitting will be required.

Plan

- ▶ This time I plan to give some examples of knitted mathematical objects.
- ▶ No previous knowledge of knitting will be required.
- ▶ It's mostly just an excuse to show off some things I've knitted.

Introduction

Universal Cover Scarf

Mathematical background

Knitting background

Finished object

Seifert surfaces

Mathematical background

Finished object

Cellular Automata shawl

Mathematical background

Knitting background

Finished object

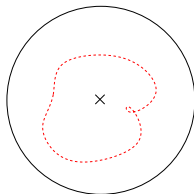
Hearts

Universal covers

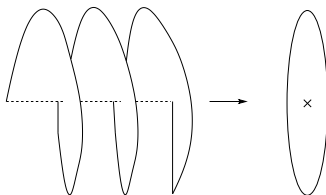
- ▶ The idea is to take a topological space X , which may not be simply connected (that is, contains loops which can't be contracted), and construct a new space which closely resembles X , but is simply connected.
- ▶ This is done by taking copies of small patches of X and gluing them together in such a way that any closed curve in X which can't be contracted to a point returns to a different copy of that point than it started at, and hence is not closed.

The punctured disc

- Take a disc D^2 , and remove the origin.



- This space is not simply connected, and its universal cover looks like



Metrics

- ▶ Before we knit this, we will need to put a *metric* on the space.
- ▶ If we use the normal Euclidean metric, as we approach the centre of the disc, the length of the universal cover approaches 0, so our scarf would have zero length. Instead we use the metric suggested by the previous picture.
- ▶ The rows are then helicoids, which have the (surprisingly uncommon) feature that you can actually calculate their lengths.

$$L = \int_0^1 \sqrt{a^2 + b^2} dt = \sqrt{a^2 + b^2}$$

Knitting homogeneous spaces

- ▶ To knit a homogeneous space, all you need to do is:
 - ▶ Calculate the length of each row
 - ▶ Work out how many stitches shorter the next row is
 - ▶ Place 'decrease' stitches as evenly as possible along the row

Universal cover scarf



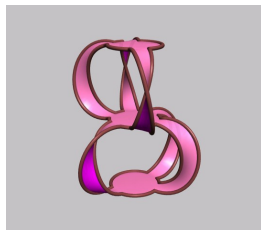
Scarf-generating program

- ▶ The pattern for this scarf is computer-generated, so we can use it to calculate patterns with different parameters.
- ▶ The program will ask for
 - ▶ The length of the scarf in stitches.
 - ▶ The ratio of the height of a stitch to its width.
 - ▶ The width of the scarf in rows.

It will prompt with suggested values for these parameters as a guide.

What is a Seifert surface?

- ▶ Given any knot, it is possible to find an oriented surface in \mathbb{R}^3 of which this knot is the boundary.
- ▶ This can be proved constructively, by giving an algorithm to construct such a surface.
- ▶ This algorithm, *Seifert's algorithm* expresses the surface as a series of discs linked by twisted bands:



How to knit a Seifert surface

Then to knit a Seifert surface, you just need to know

- ▶ How to knit a disc
- ▶ How to knit a twisted band
- ▶ How to connect these components together.

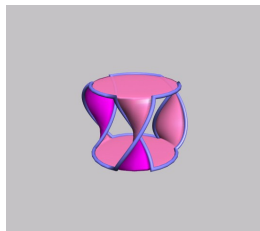
Trefoils knots

Then to knit a Seifert surface of a trefoil:

- ▶ Start at the centre of one disc, and knit outwards to form that disc.
- ▶ Divide the boundary into three sections, and from each one knit a band. To make this band twist, 'short rows' are used – you knit the stitches at the edge more times than those in the middle.

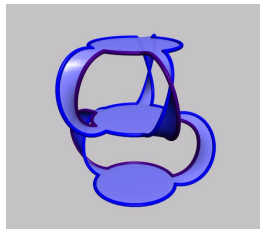
Trefoil knots (continued)

- ▶ When all three bands are complete, twist them as desired and join them all together to make the boundary of the second disc, before working inwards to fill this disc in.
- ▶ Crocheting around the edge of the surface in a different colour highlights the trefoil knot.



Seifert surfaces of general knots

- ▶ The program SeifertView can give a Seifert surface of a general knot in this form, as a series of discs linked by twisted bands.
- ▶ It's slightly more tricky, but using this kind of construction it's possible to give an explicit pattern for knitting any Seifert surface of this form.



Seifert surfaces



Seifert earrings

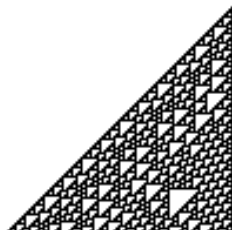


Cellular automata

- ▶ (One dimensional) Cellular automata are rules for generating patterns of binary digits.
- ▶ They take a binary string and create a new binary string, with each value determined by its previous value and its two neighbours. Then taking all of these rows together gives a two-dimensional binary pattern.
- ▶ Cellular automata appear in any dimension; Conway's "Game of life" is a two-dimensional example.

Cellular automata (continued)

- ▶ Despite the simplicity of these rules, their behaviour can be very complicated. One rule (“rule 110”) is Turing complete, which means that it is possible to construct a universal computer using just these binary strings.
- ▶ They can also produce striking and chaotic patterns:



Lace knitting

- ▶ Lace knitting is “making holes in your knitting on purpose” (yarnover.net).
- ▶ One simple technique in lace knitting is the “yarn over” stitch. This adds a stitch, which appears as a small hole. By pairing this with a decrease stitch we can end up with the same number of stitches overall.
- ▶ This then gives us a way to represent the Cellular Automaton pattern - with a 1 representing a hole, and a 0 a stitch knitted plainly.

Lace knitting (continued)

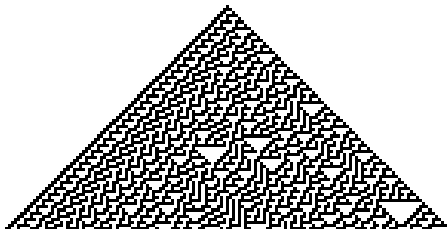
- ▶ There are, however, several kinds of decrease stitch we could use, in particular there are “left-leaning” and “right-leaning” decreases, which form neat rows when arranged in left- and right-ward travelling columns respectively.
- ▶ To choose which type to use, we can go through the Cellular Automaton pattern and assign each ‘1’ to a chain with a fixed direction.



Cellular automata shawl

This is the motivation behind the Cellular automata shawl:

- ▶ Starting from a single 1 in a row of 0s, a pattern is generated using Rule 30:



- ▶ A second program then reads through the binary pattern, and interprets the binary digits as chains of left-leaning, right-leaning and stationary chains.
- ▶ These are then converted into a lace pattern, and knit as a triangular shawl.

Cellular automata shawl

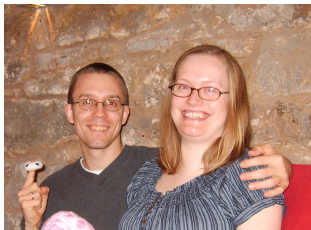


CA shawl program

- ▶ The CA shawl program will take a string of binary digits and use this as the basis for a lace shawl pattern. It does this in a number of steps:
 - ▶ Reads this string in.
 - ▶ Generates a CA pattern (using 'Rule 30').
 - ▶ Interprets this as a collection of chains.
 - ▶ Then converts this into a ready-to-knit lace pattern.

Juliana

- ▶ Juliana was a friend from our knitting group.
- ▶ She suffered from heart problems since birth, but this took a turn for the worse a few years ago, which led to her being put on the heart transplant waiting list.
- ▶ Last autumn she died of heart failure, following a long wait for a transplant.



- ▶ We decided it would be a fitting tribute to her to knit and give away stuffed hearts.

Organ donation

- ▶ I'd also like to give out some information about organ donation – I have books from Legacy of Life, a local(ish) charity who tell the stories of people who have received transplants.
- ▶ So, if I can give you one of the hearts or a Legacy of Life book let me know!



Questionable Content, “Large hadron crocheter”



Thanks for listening! Also thanks to:

- ▶ Glamorous Assistant Pam!
- ▶ Florian and Thomas on Tech Support!